

# **Distribuované systémy a výpočty**

**X36DSV**

**Jan Janeček**  
(dnes Peter Macejko)



# Výlučný přístup

## Algoritmy na úplném grafu

### Lamport

- základní algoritmus,  $3(n-1)$  zpráv/požadavek

### Ricart – Agarwala

- pozdržení souhlasů,  $2(n-1)$  zpráv/požadavek

### Carvalho – Roucairol

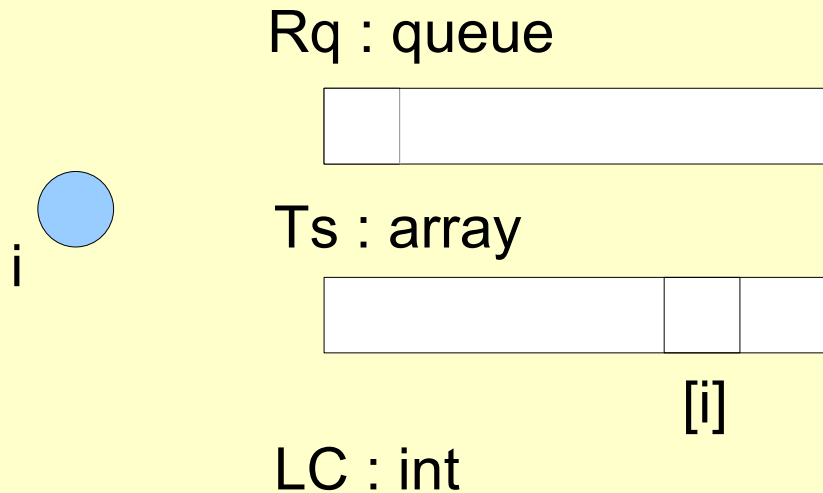
- kredity pro přístup,  $0 - 2(n-1)$  zpráv/požadavek

### Ricart – Agarwala

- požadavkový token,  $n$  zpráv/požadavek



# Lamport



```
{ initialization }  
begin  
  LC := 0;  
  for j := 1 to N do  
    begin  
      Ts[j] := 0; Rq[j] := ∞  
    end  
  end  
end
```

## Legenda:

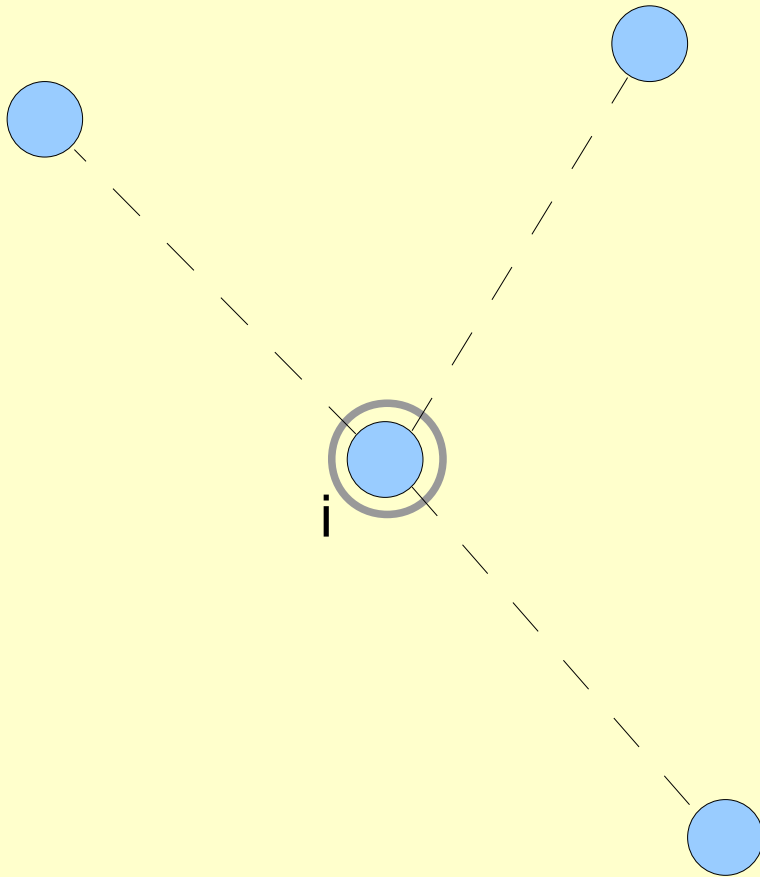
LC – local clock (lokální čas)

Rq – request queue (prioritní fronta)

Ts – time stamps (čas poslední zprávy)



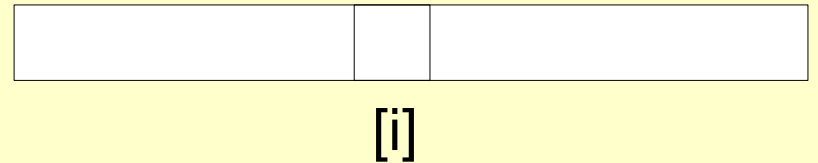
# Lamport



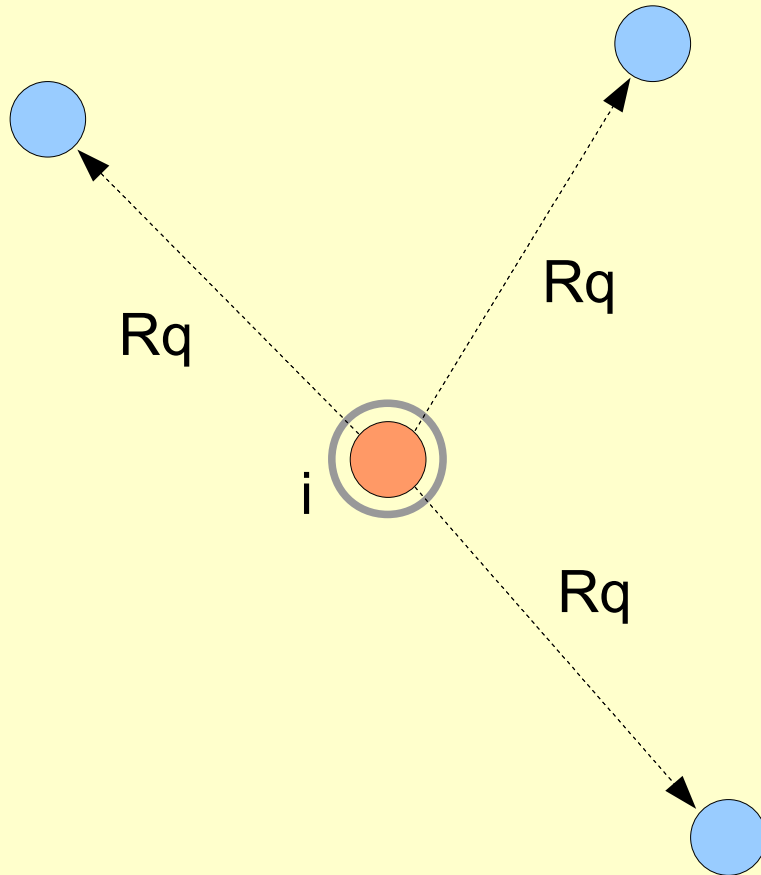
Rq : queue



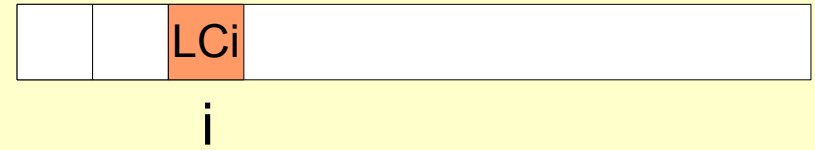
Ts : array



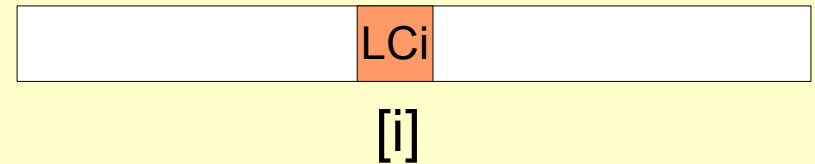
# Lamport



Rq : queue



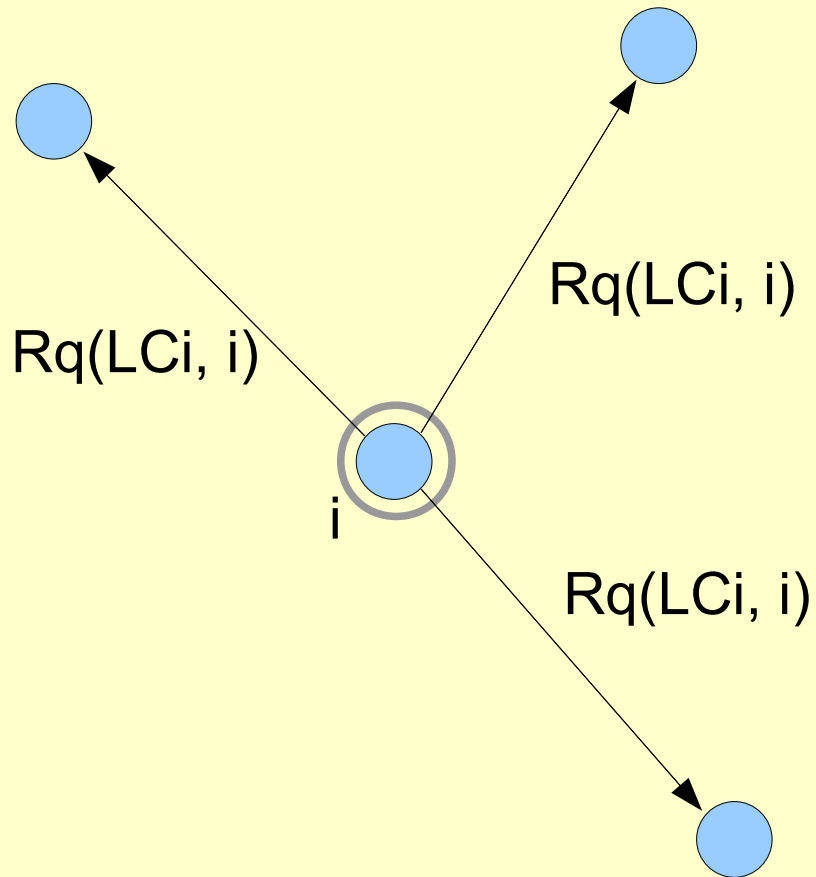
Ts : array



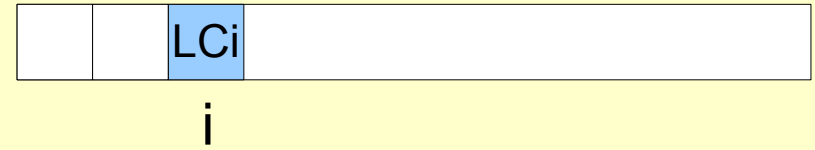
Odesláno (n-1) zpráv



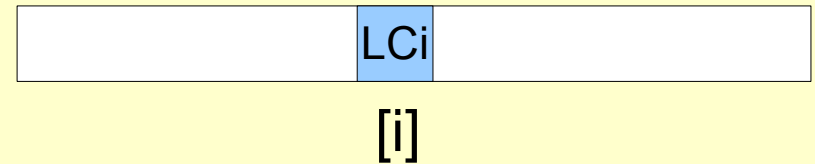
# Lamport



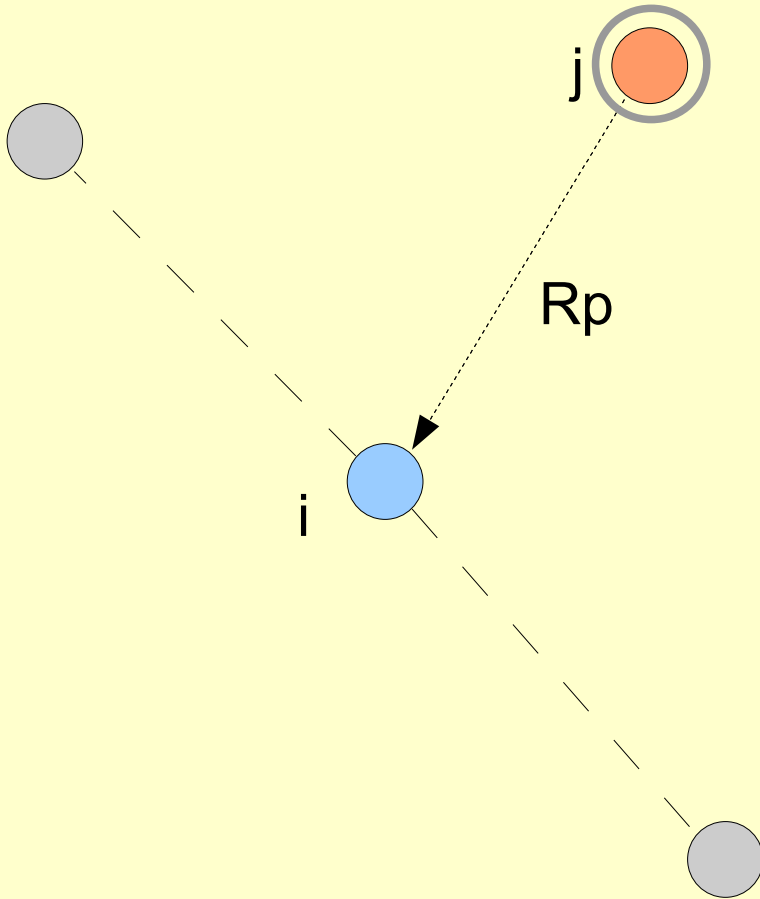
Rq : queue



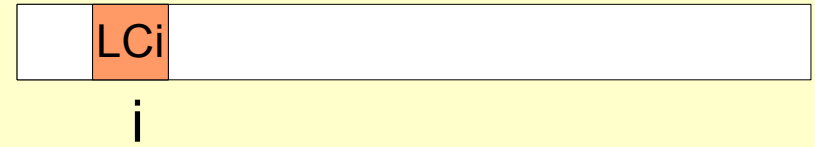
Ts : array



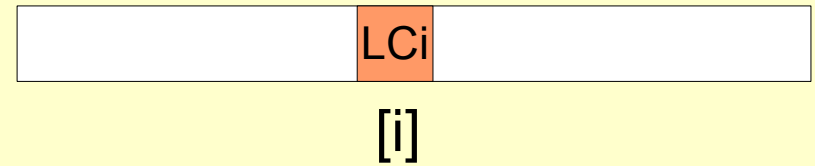
# Lamport



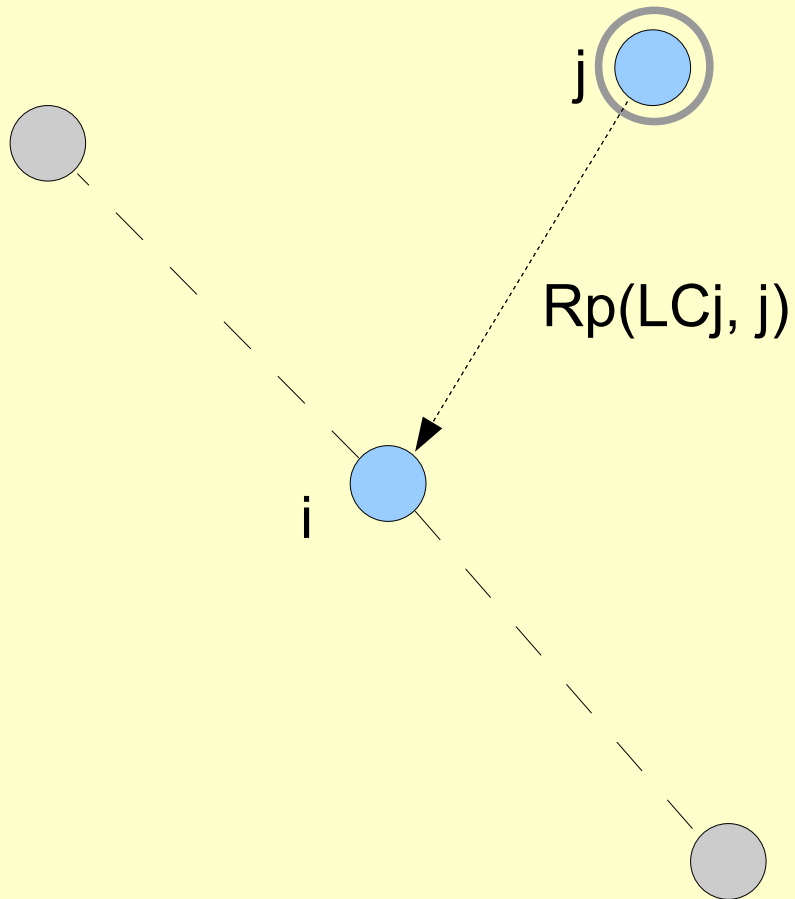
Rq : queue



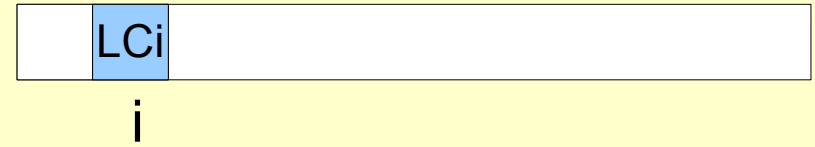
Ts : array



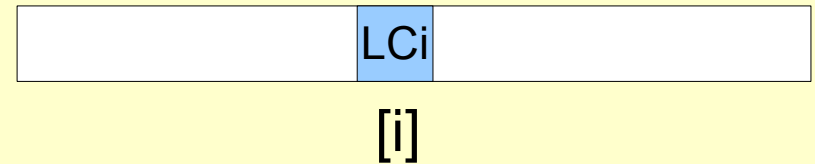
# Lamport



Rq : queue

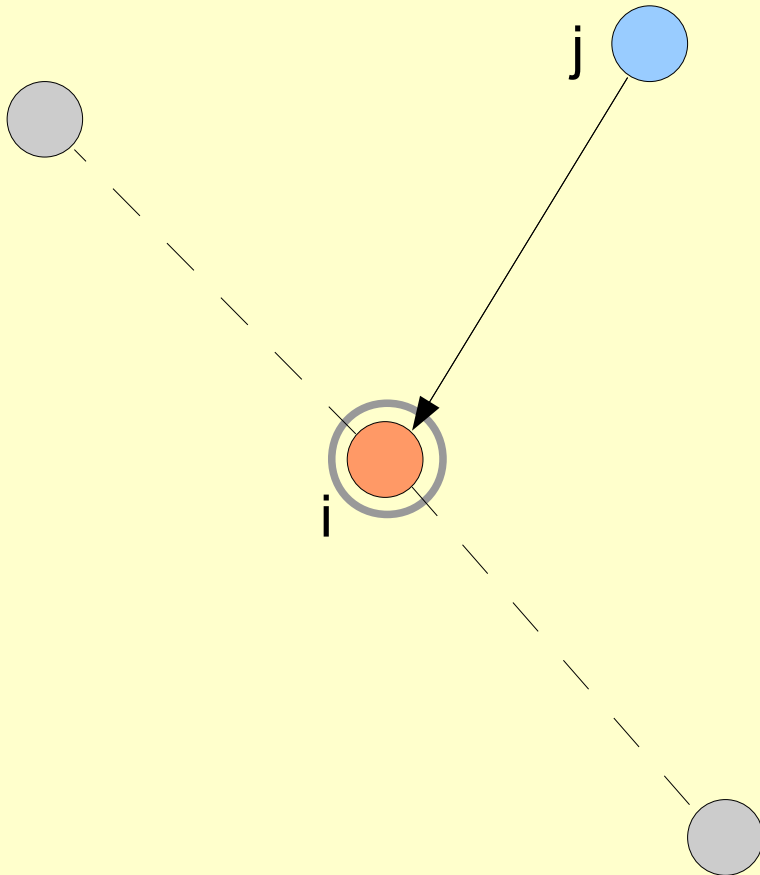


Ts : array

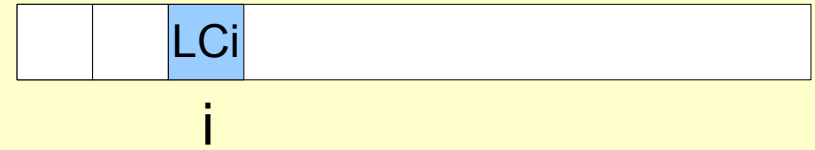




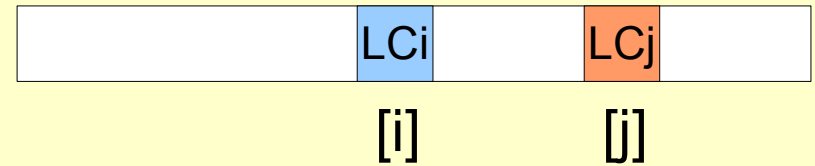
# Lamport



Rq : queue



Ts : array



Přijato (n-1) zpráv



# Lamport

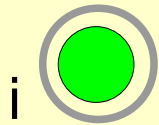
```
when request { access request }  
begin  
  [P] Rq[i] := LC; Ts[i] := LC; LC := LC+1; [V]  
  for j:=1 to N do  
    if j≠i then  
      send REQUEST(LC,i) to j
```

```
when received REQUEST(ts,j) { j-th process request }  
begin  
  [P] LC := max(LC,ts); LC := LC+1; [V]  
  Rq[j] := ts; Ts[j] := ts;  
  send RESPONSE(LC,i) to j  
end
```

```
when received RESPONSE(ts,j) { j-th process response }  
begin  
  [P] LC := max(LC,ts); LC := LC+1; [V]  
  Ts[j] := ts  
end
```



# Lamport



Rq : queue



i

Ts : array

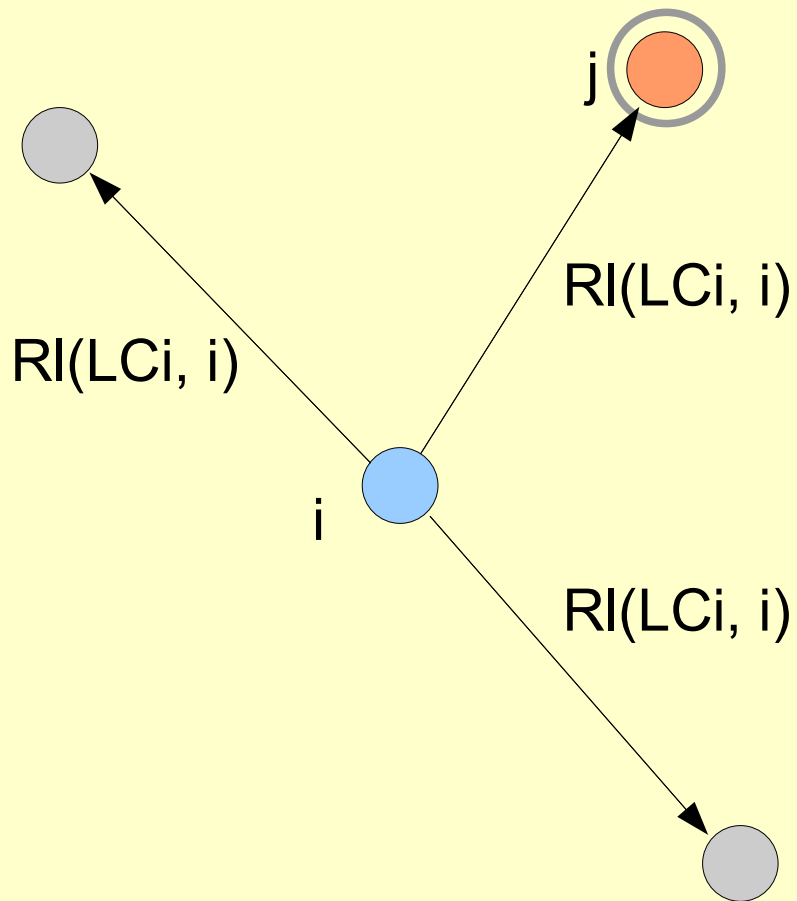


[i]

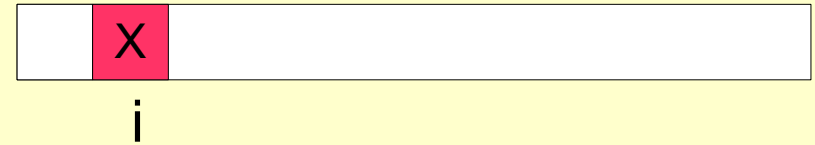
X < Y



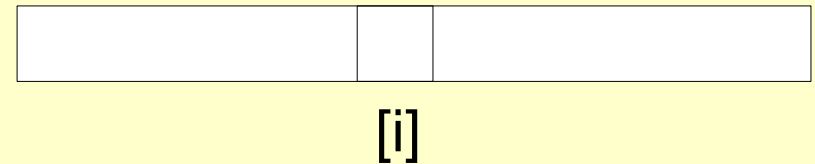
# Lamport



Rq : queue



Ts : array



Odesláno  $(n-1)$  zpráv



# Lamport

```
when (Rq[i]<Rq[j] forall j≠i) and (Rq[i]<Ts[j] forall j≠i)
  begin
```

```
    { critical section }
```

```
    send RELEASE(LC) to j
  end
```

```
when received RELEASE(ts,j)
  begin
```

```
{ j-th process release }
```

```
    [P] LC := max(LC,ts); LC := LC+1; [V]
    Rq[j] := ∞;
```

```
end
```

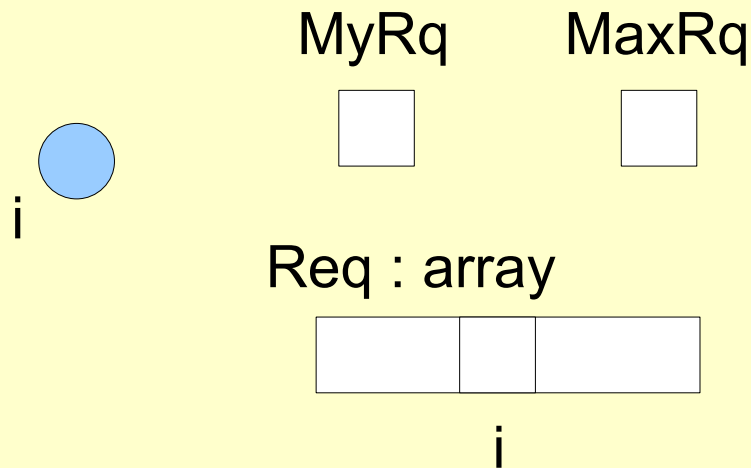
## Zprávy

request + response + release

$$(n-1) + (n-1) + (n-1) = 3x(n-1)$$



# Ricart - Agarwala



**{ initialization }**

**begin**

    MaxRq:=0; MyReq:=F;

**for** j:=1 **to** N **do**

        Req[j]:=F

**end**

Legenda:

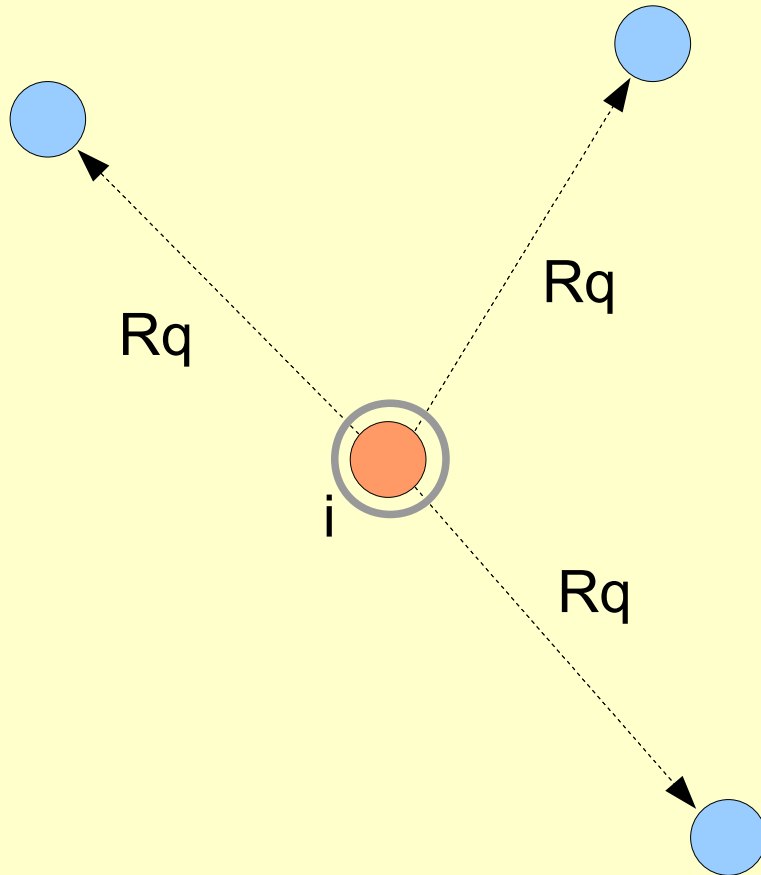
MyRq – sekvenční číslo vlastní žádosti

MaxRq – maximální sekvenční číslo

Req – pole registrací žádostí na vstup do CS



# Ricart - Agarwala



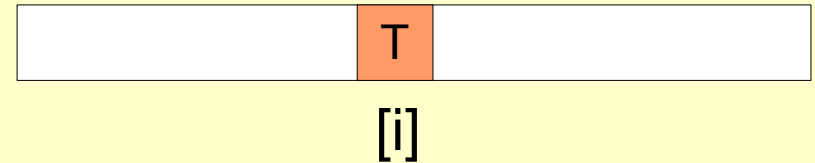
MyRq

S+1

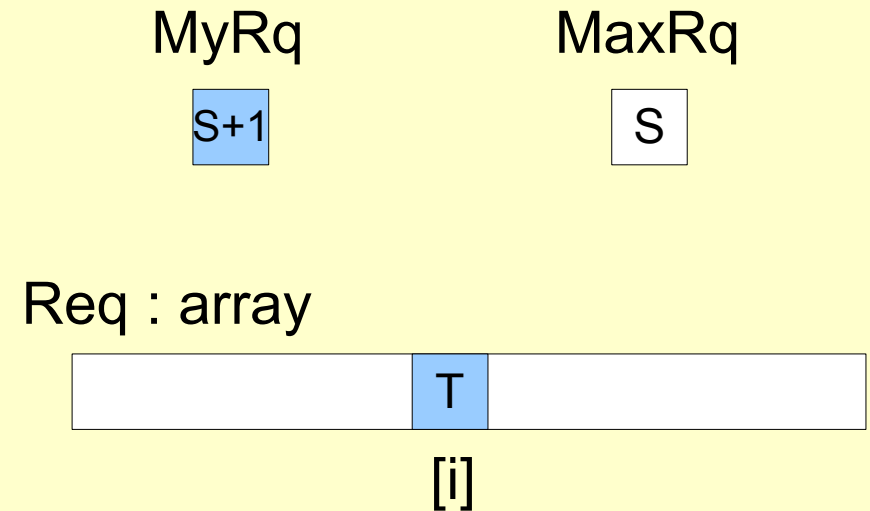
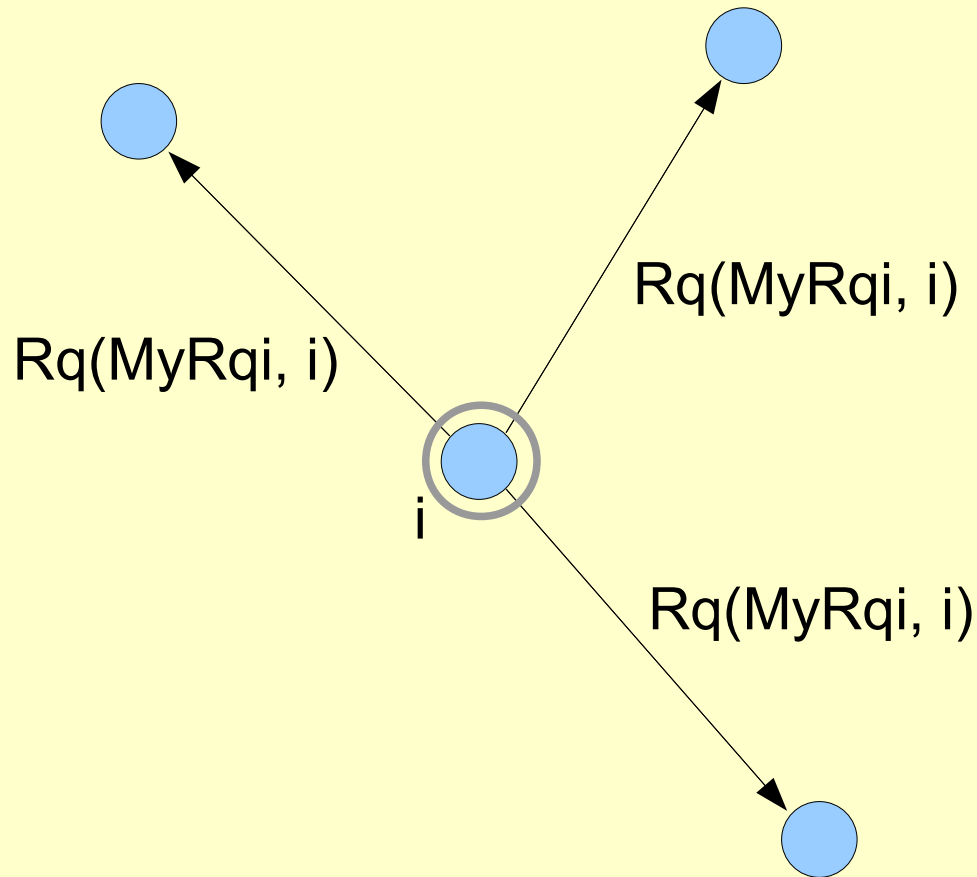
MaxRq

S

Req : array



# Ricart - Agarwala

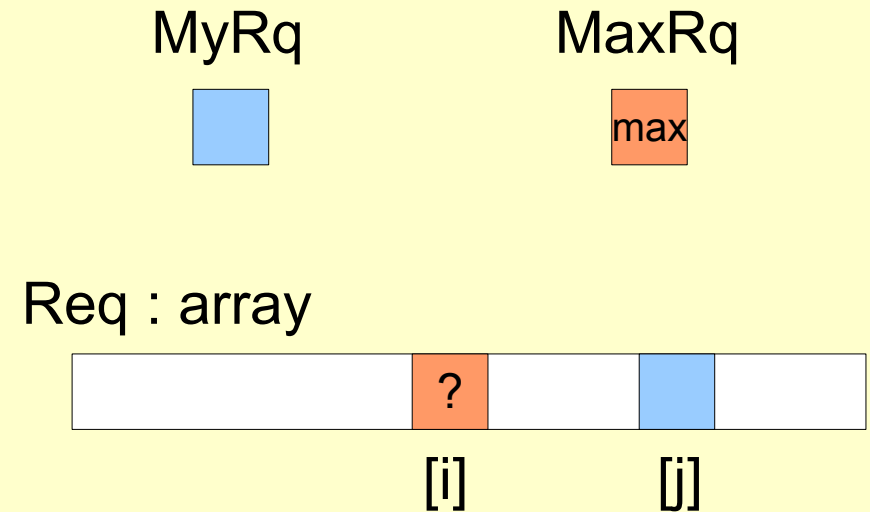
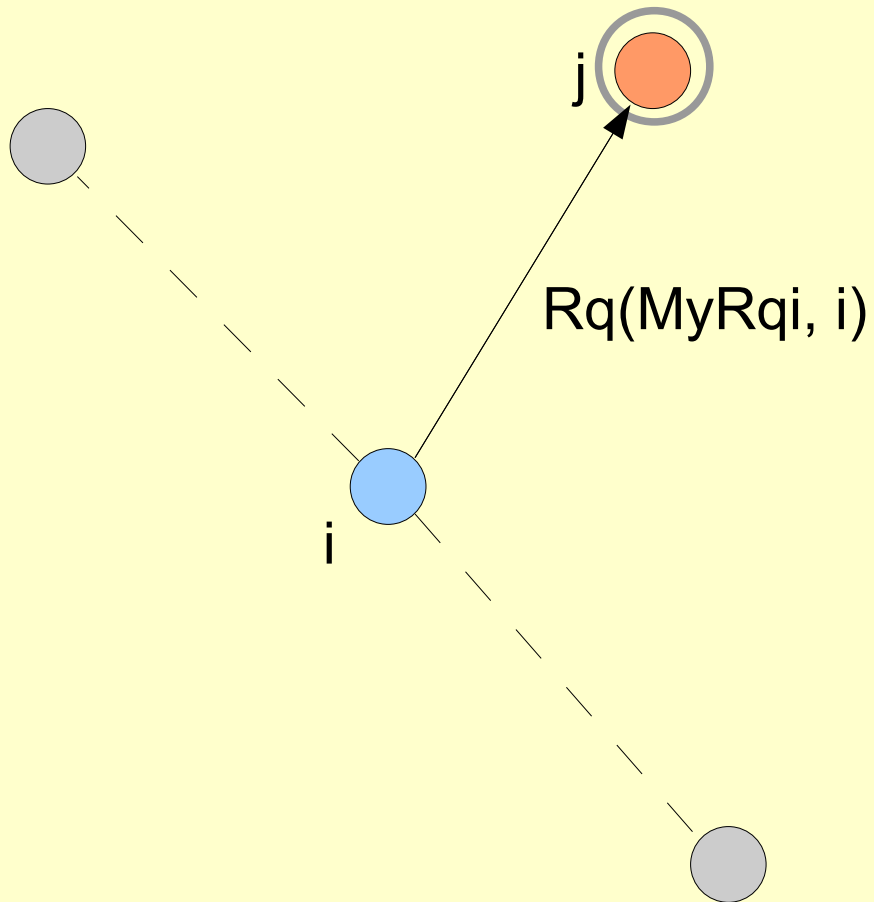


Odesláno  $(n-1)$  zpráv

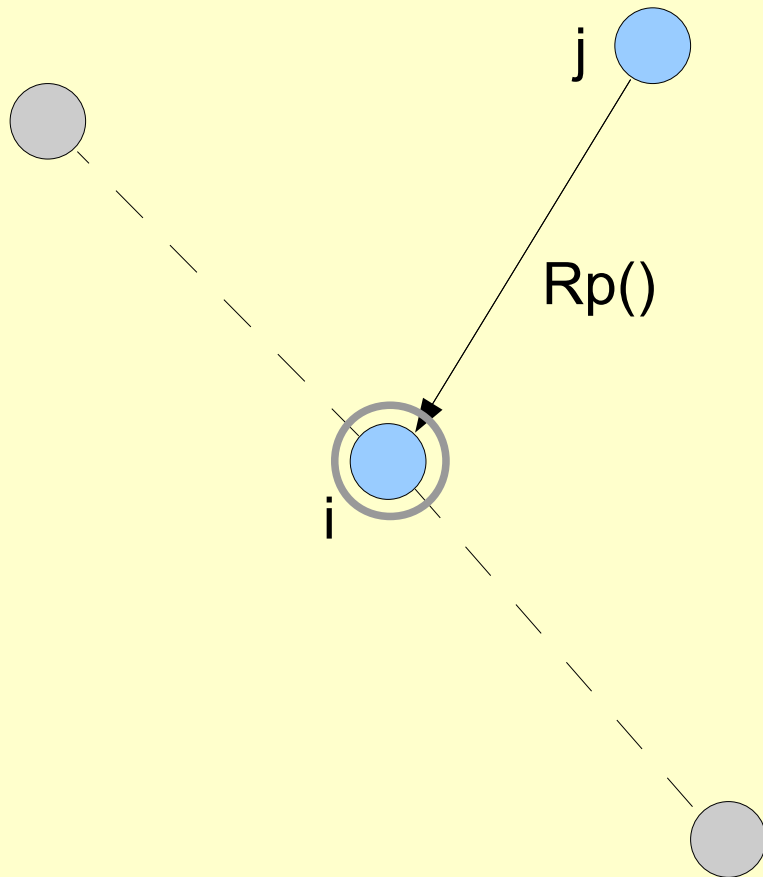




# Ricart - Agarwala



# Ricart - Agarwala



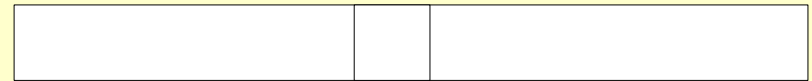
MyRq



MaxRq



Req : array

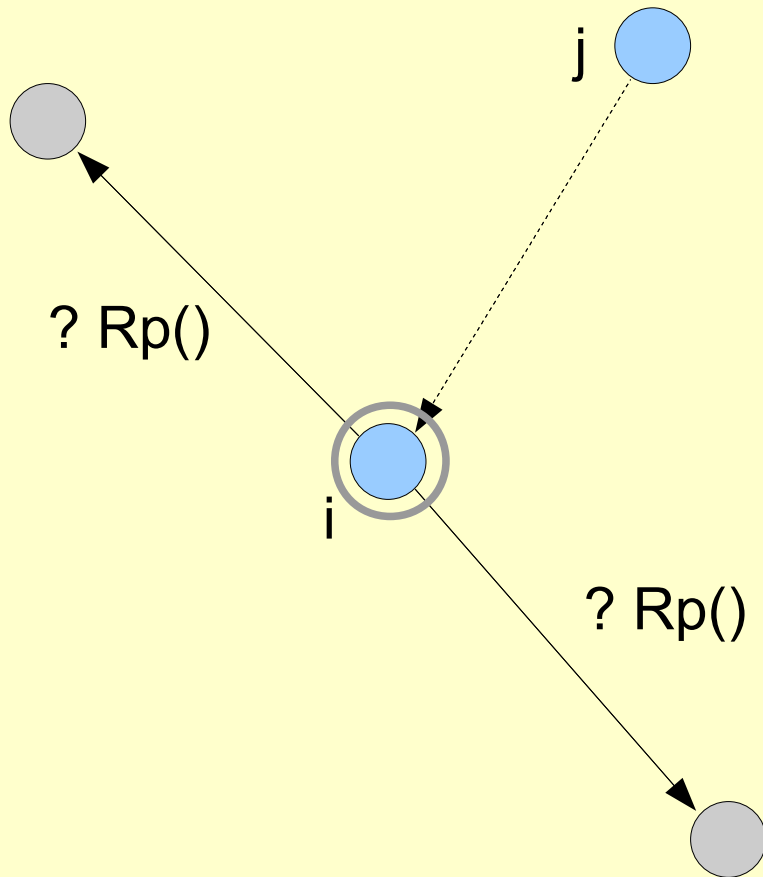


$[i]$

Přijato (n-1) zpráv



# Ricart - Agarwala



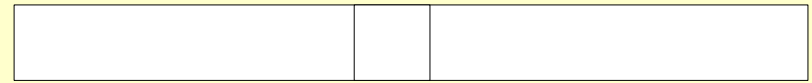
MyRq



MaxRq



Req : array



[ $i$ ]



# Ricart - Agarwala

```
when request                                     { access request }
  begin
    [P] Req[i] := T; MyRq := MaxRq+1; [V]
    RpCnt := 0;
    for j:=1 to N do
      if j≠i then
        send REQUEST(MyRq,i) to j;
    wait RpCnt=N-1;
    { critical section }
    Req[i] := F;
    for j:=1 to N do                               { delayed responses }
      if Req[j] then
        begin
          Req[j]:=F;
          send REPLY to j
        end
    end
  end
```

end



# Ricart - Agarwala

```
when received REQUEST(k,j) do           { request of the k-th process }
  begin
    MaxRq := max(MaxRq,k);
    [P] Delay := Req[i] and ((k>MyRq) or (k=MyRq and j>i)); [V]
    if Delay then
      Req[j] := T
    else
      send REPLY to j
    end
  end

when received REPLY do                 { response of any process }
  RpCnt:=RpCnt+1;
```

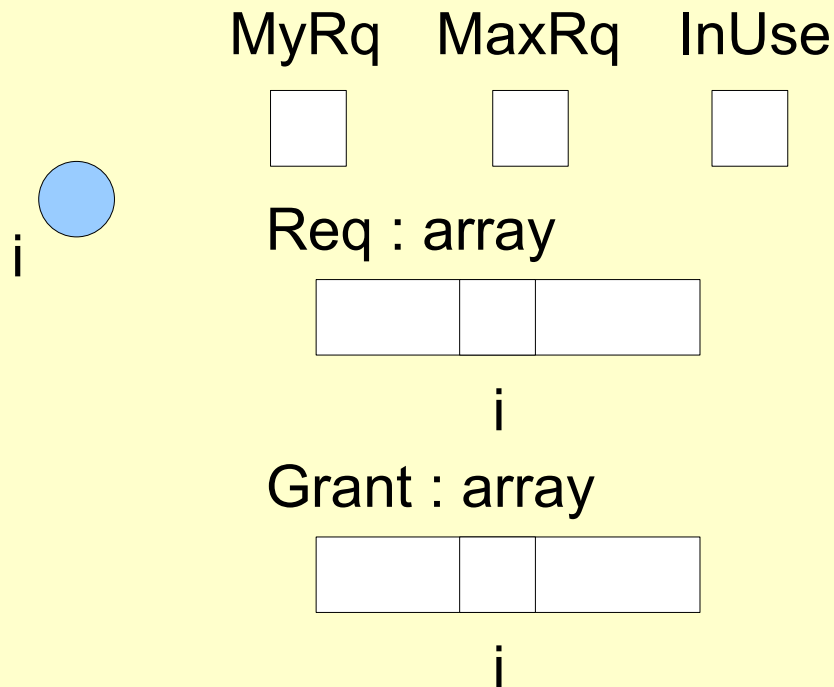
## Zprávy

request + response

$$(n-1) + (n-1) = 2x(n-1)$$



# Carvalho - Roucairol



**{ initialization }**

**begin**

MaxRq := 0; MyRq := 0;

**for** j:=1 **to** N **do**

**begin**

Req[j] := F;

Grant[j] := F

**end**

**end**

Legenda:

MyRq – sekvenční číslo vlastní žádosti

MaxRq – maximální sekvenční číslo

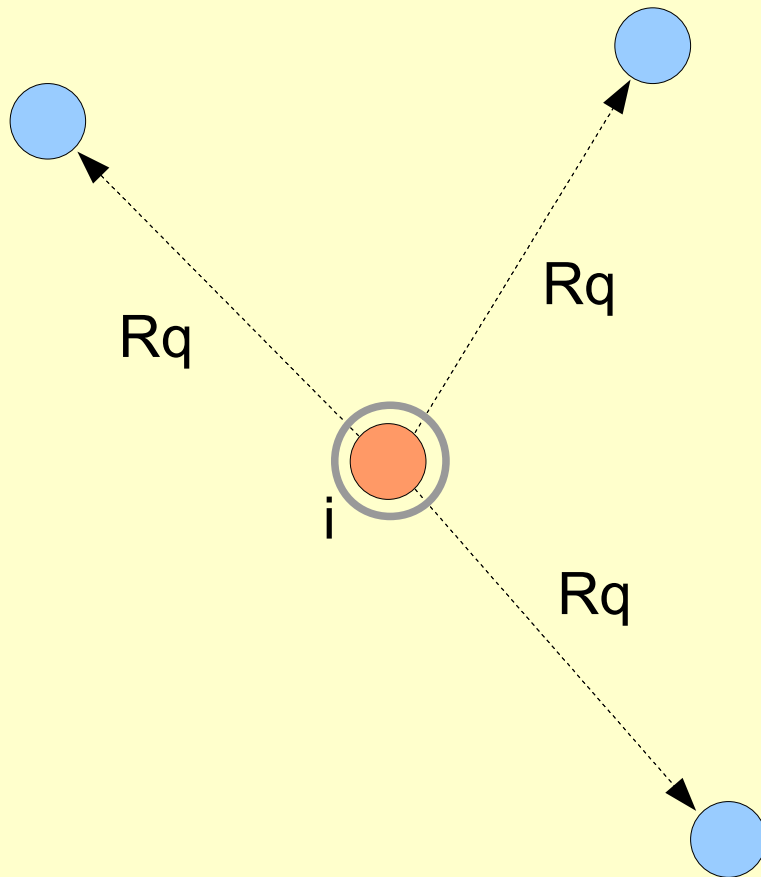
InUse – identifikátor kritické sekce

Req – pole registrací žádostí na vstup do CS

Grant – pole aktivních 'pověření'



# Carvalho - Roucairol



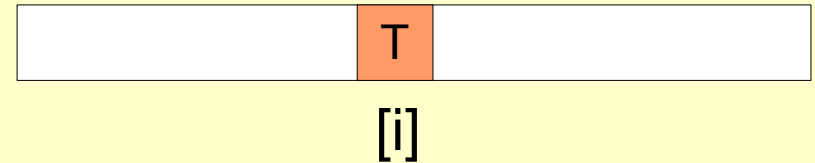
MyRq

S+1

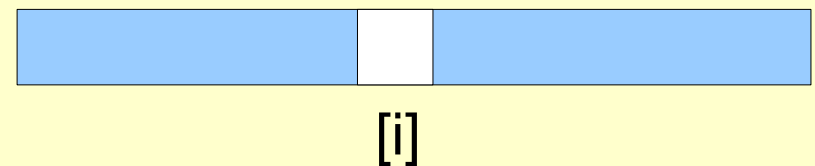
MaxRq

S

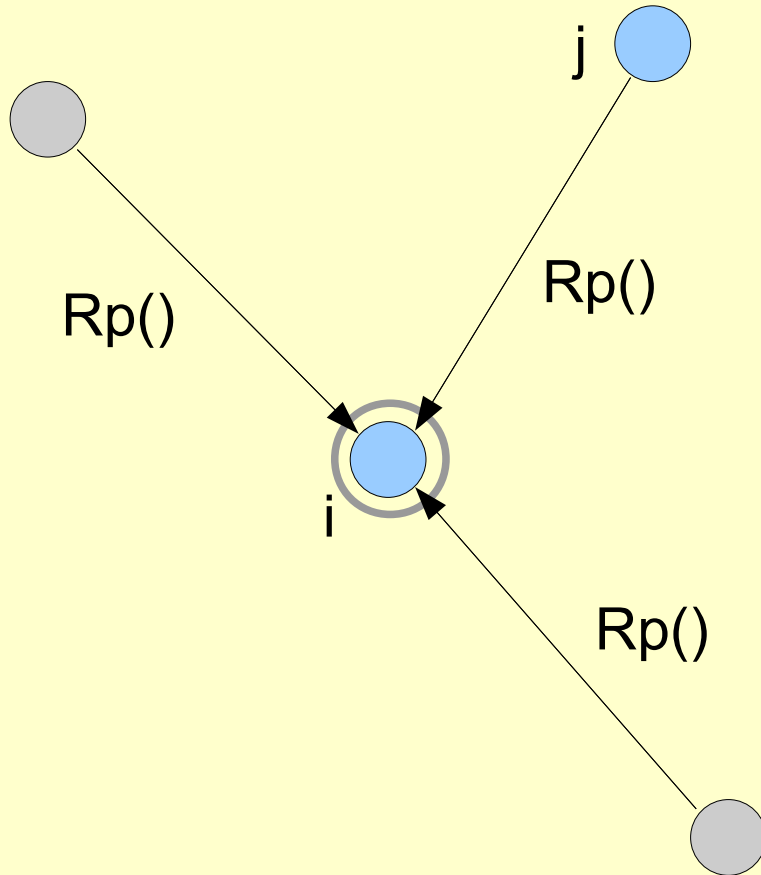
Req : array



Grant : array;



# Carvalho - Roucairol



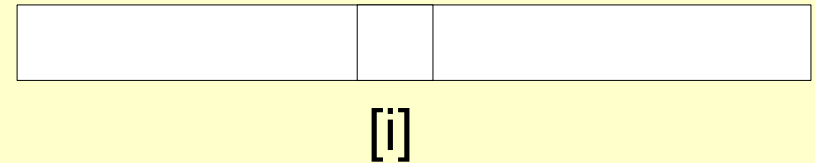
MyRq



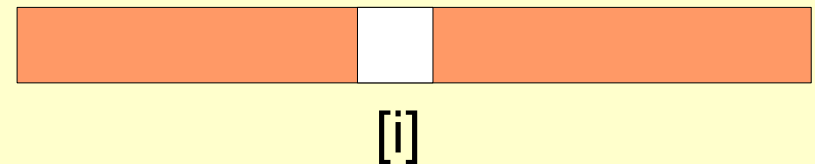
MaxRq



Req : array



Grant : array;





# Carvalho - Roucairol

```
when request { access request }  
  [P] Req[i] := T; MyRq := MaxRq+1; [V]  
  for j:=1 to N do  
    if j≠i and (not Grant[j]) then  
      send REQUEST(MyRq,i) to j;  
  wait (Grant[j]=T forall j≠i);  
  Req[i] := F; InUse := T;
```

{ critical section }

```
InUse := F;  
for j:=1 to N do { delayed responses }  
  if Req[j] then  
    begin  
      Grant[j] := F; Req[j] := F;  
      send REPLY to j  
    end
```



# Carvalho - Roucairol

```
when received REQUEST(k,j) do      { j-th process request }
  begin
    MaxRq := max(MaxRq,k);
    [P] Delay := ((k>MyRq) or (k=MyRq and j>i)) [V]
    if InUse or (Req[i] and Delay) then
      Req[j]:=T;
    if not (InUse or Req[i]) or
      (Req[i] and (not Grant[j]) and (not Delay)) then
      send REPLY(i) to j;
    if (Req[i] and Grant[j] and (not Delay)) then
      begin
        Grant[j]:=F;
        send REPLY(i) to j;
        send REQUEST(MyRq,i) to j
      end
    end
  end
```

end



# Carvalho - Roucairol

when received REPLY from j do      { j-th process response }  
    Grant[j] := T

## Zprávy

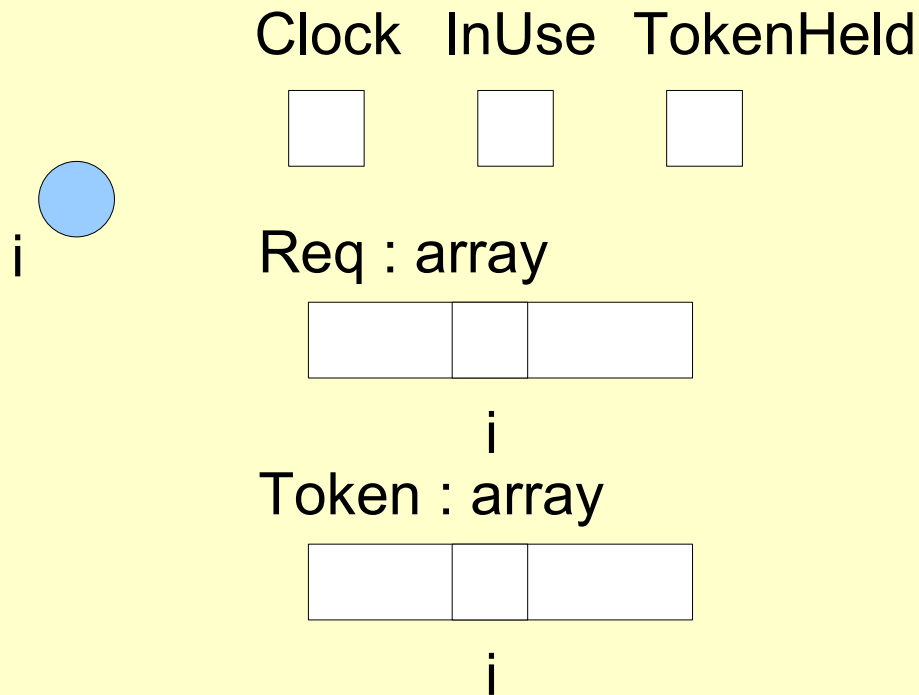
request + response

$$(n-1) + (n-1) = 2x(n-1)$$

$$0 + 0 = 0$$



# Ricart - Agarwala (token passing)



**{ initialization }**

**begin**

**for j:=1 to N do**

Req[j] := 0; Token[j] := 0;

Clock := 0;

InUse := F;

TokenHeld := (myID == 0)?T:F

**end**

Legenda:

Clock – sekvenční číslo

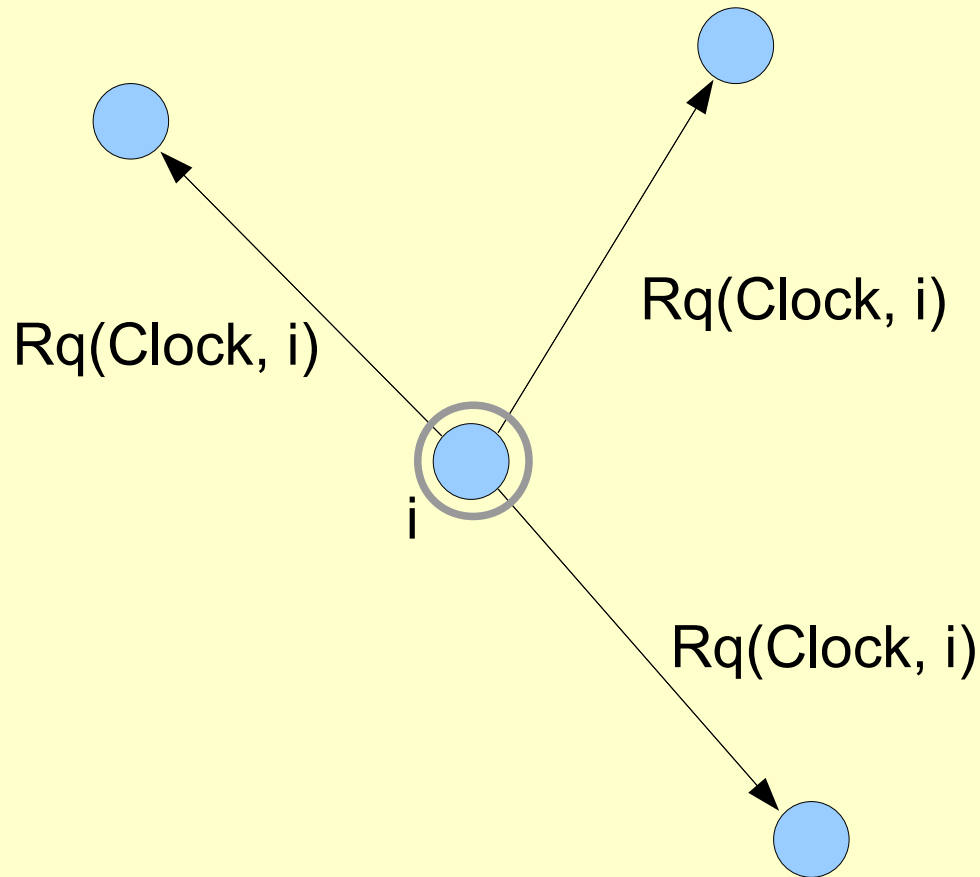
InUse – identifikátor kritické sekce

Req – pole registrací žádostí na vstup do CS

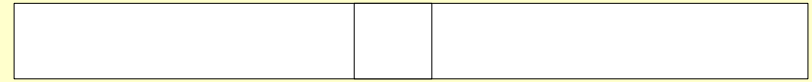
TokenHeld – identifikátor přítomnosti tokenu



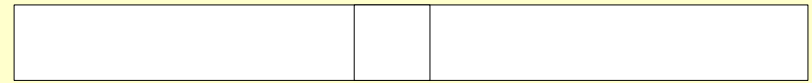
# Ricart - Agarwala (token passing)



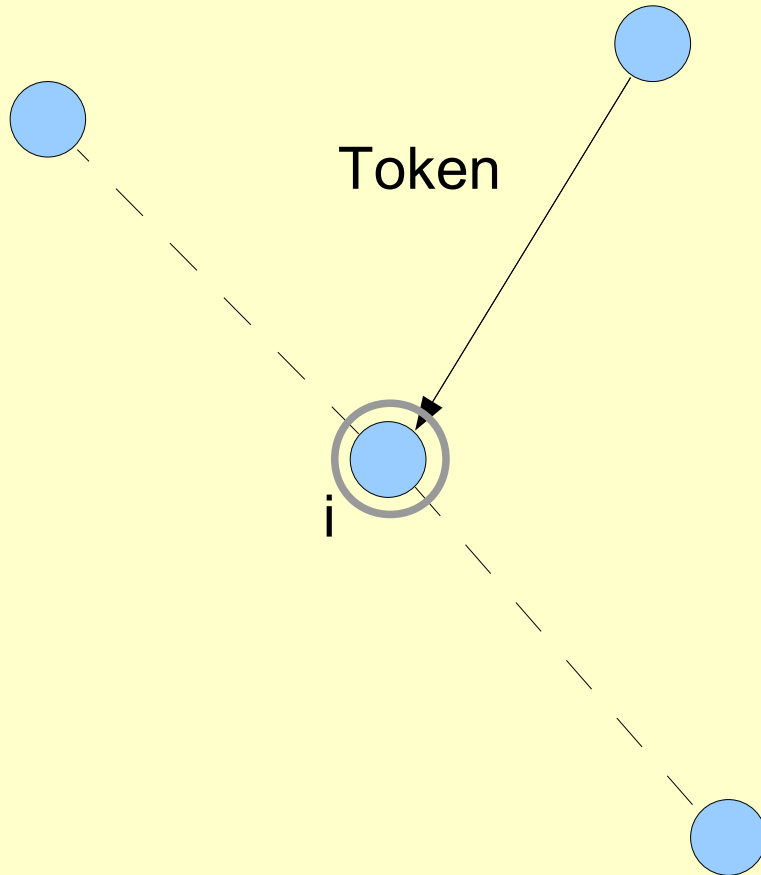
Req : array;



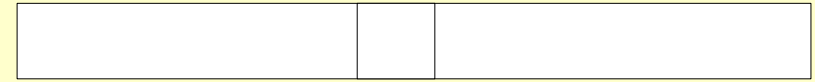
Token : array;



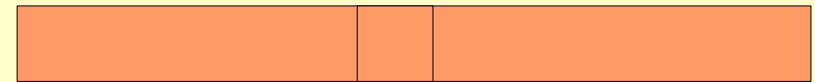
# Ricart - Agarwala (token passing)



Req : array;



Token : array;



# Ricart - Agarwala (token passing)

```
when request do
    if not TokenHeld then
        begin
            Clock := Clock+1;
            broadcast REQUEST(Clock,i);
            receive TOKEN;
            TokenHeld := T
        end;
        InUse := true;
        { critical section }
        Token[i] := Clock;
        InUse := F;
        j := (i+1) mod N;
        while i≠j do
            begin
                if Req[j]>Token[j] and TokenHeld then
                    begin
                        TokenHeld := F; send TOKEN to j;
                        j := (j+1) mod N
                    end
                end
            end
        end
```

{ access request }

{ broadcasting request }

{ waiting for token }

{ passing token }



# Ricart - Agarwala (token passing)

```
when received REQUEST(k,j) do                                     { j-th process request }
  begin
    Req[j]:=max(Req[j],k);
    if TokenHeld and not InUse then
      begin
        j:=(i+1) mod N;
        while i<>j do
          begin
            if Req[j]>Token[j] and TokenHeld then
              begin
                TokenHeld:=F; send TOKEN to j;
                j:=(j+1) mod N
              end
            end
          end
        end
      end
    end
  end
```

{ passing token }





# Výběr – leader election

## Rozbití symetrie

### Výběr na stromu

- základní algoritmus,
- vyžaduje  $3(n-1)$  zpráv

### Výběr na kruhu

#### Chang – Roberts

- jednosměrná komunikace,
- komunikační složitost  $n \cdot \log(n) < n^2$

#### Hirschberg – Sinclair

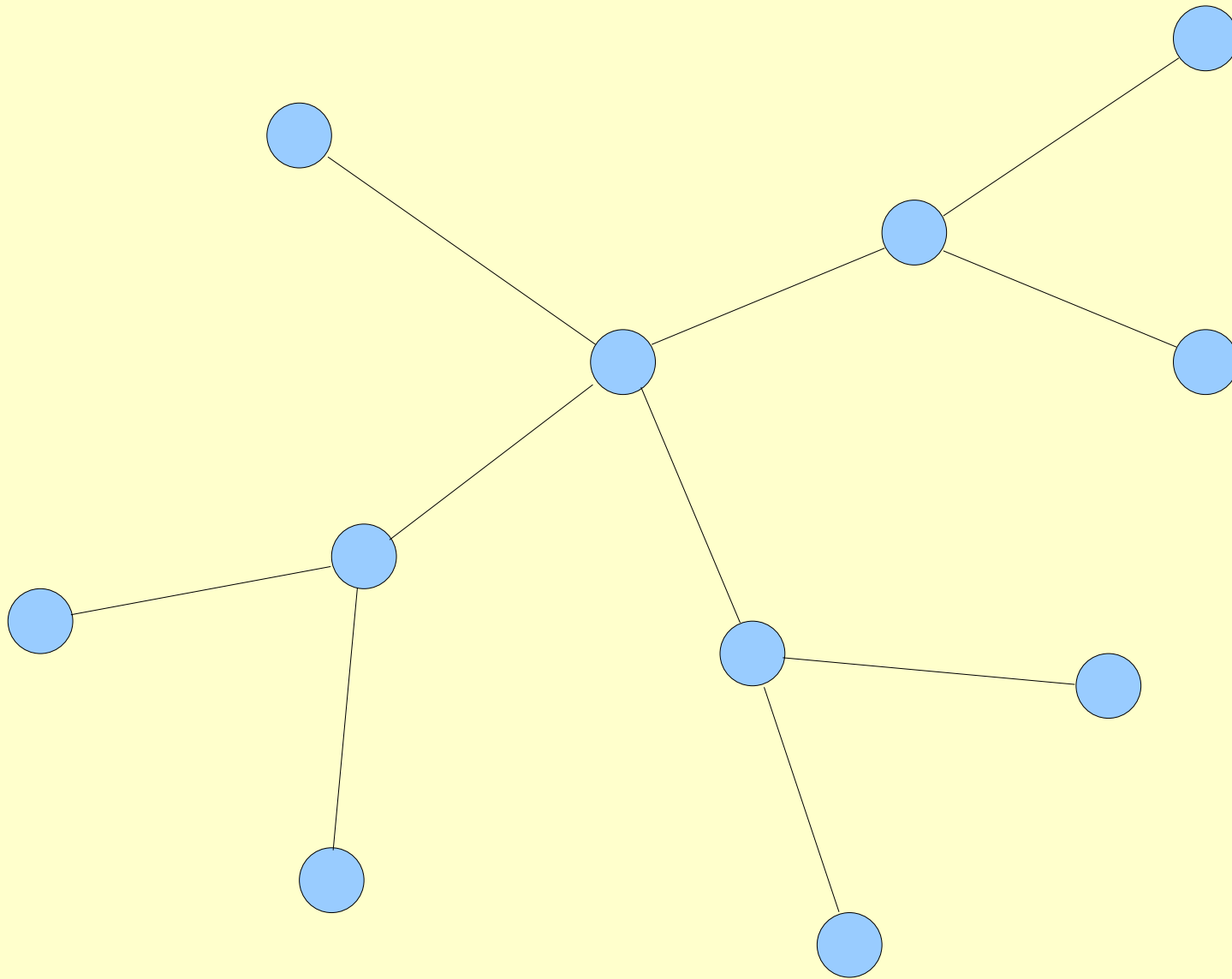
- obousměrná komunikace
- komunikační složitost  $n \cdot \log(n)$

#### Peterson/Dolev – Klave – Rodeh

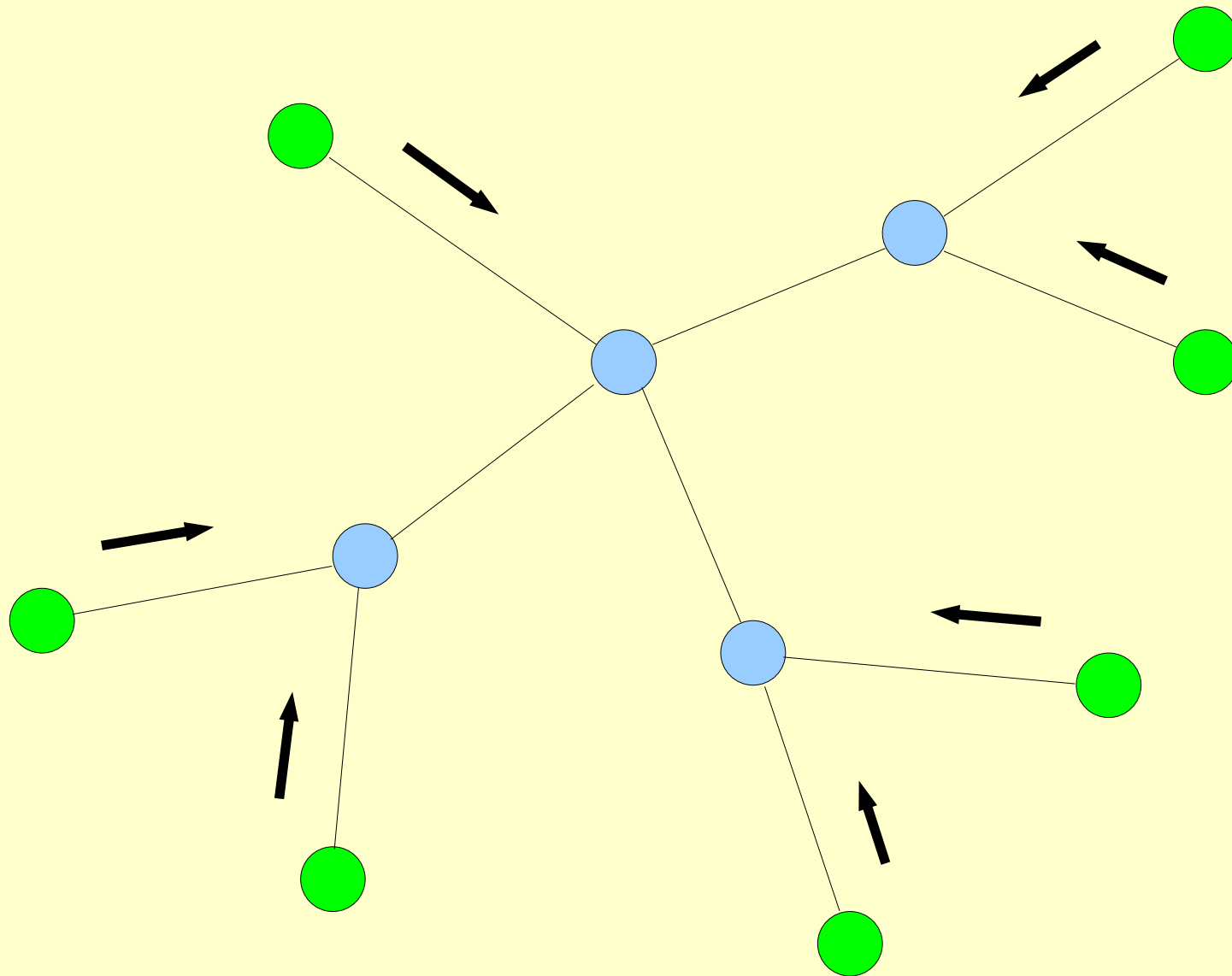
- jednosměrná komunikace,
- komunikační složitost  $n \cdot \log(n)$



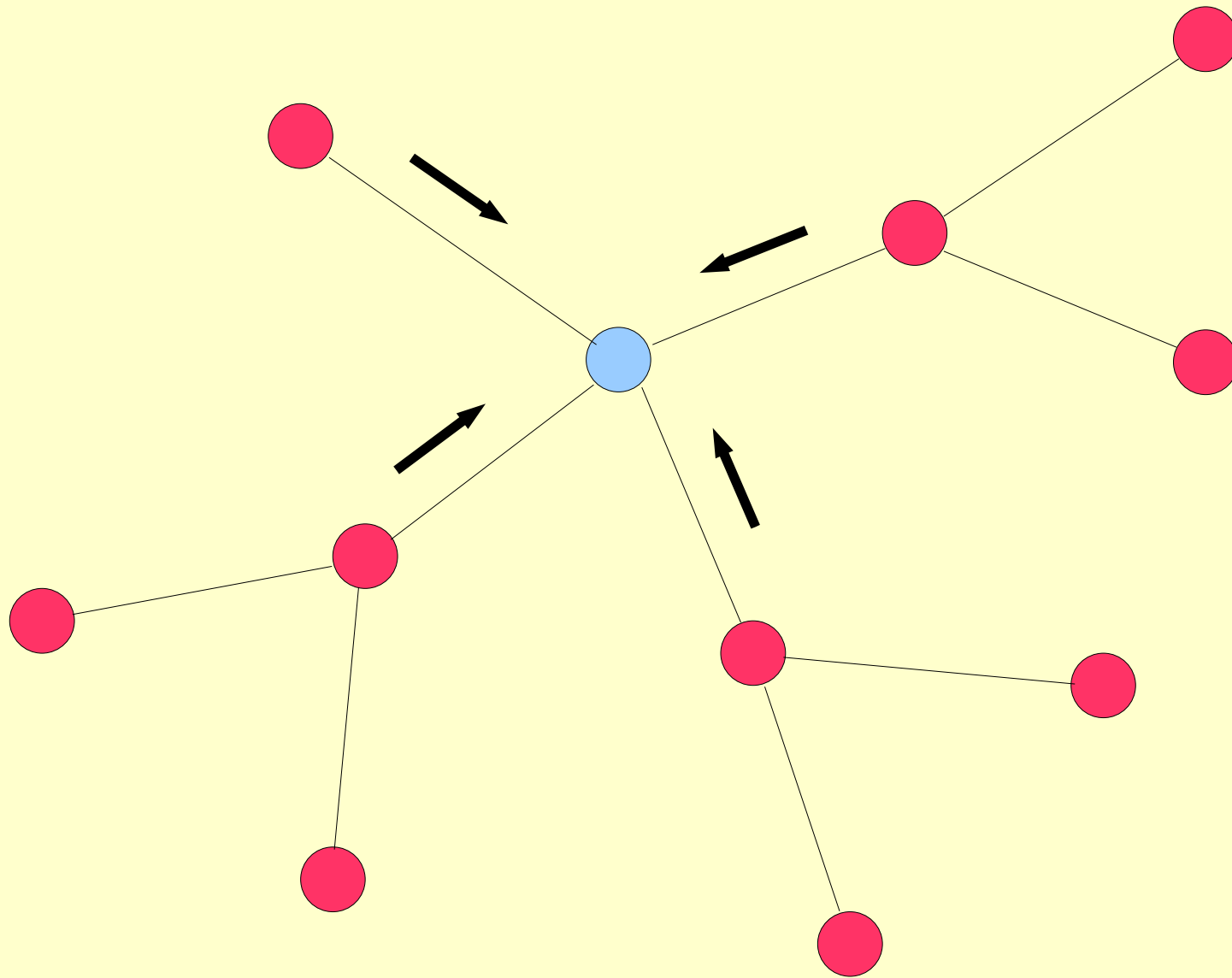
# Election on tree



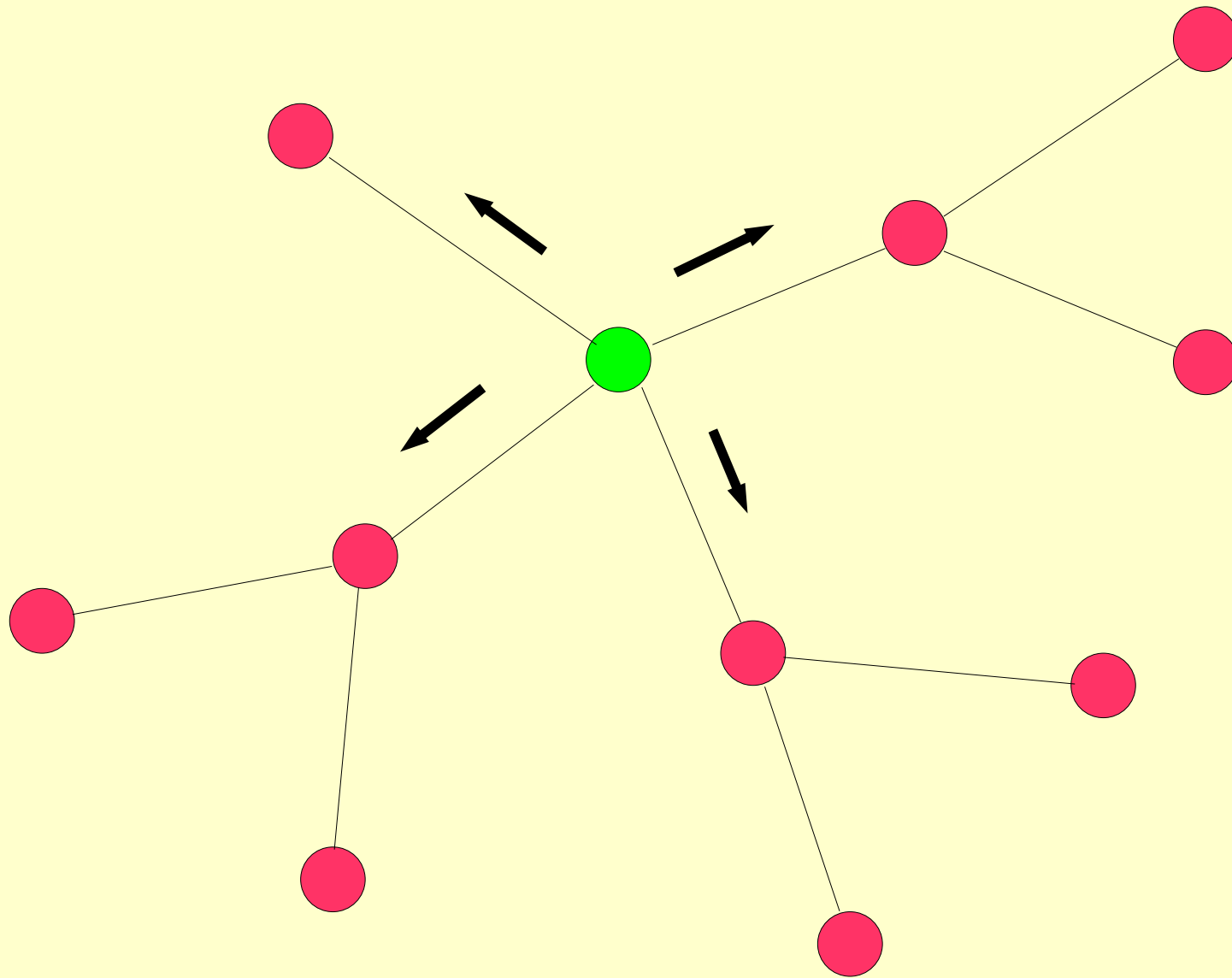
# Election on tree



# Election on tree



# Election on tree



# Election on tree

```
var  wsp :   boolean;           init false;           // wake-up sent
      wrp :   boolean;           init 0;               // wake-up received
      recp[q]: boolean;           init false;          // received from q
      vp :    P;                  init p;                 // node id
      statep : {sleep,leader,lost}; init sleep;         // node state
```

```
begin if p is initiator then
  begin wsp := true;
    forall q in Neighp do send (wakeup) to q
  end;
  while wrp < #Neighp do
    begin receive (wakeup); wrp := wrp + 1;
      if not wsp then
        begin wsp := true;
          forall q in Neighp do send (wakeup) to q
        end
      end
    end;
end;
```



# Election on tree

```
/* start of the tree algorithm */
while #{q : ~recp[q]} > 1 do
    begin receive(tok,r) from q; recp[q] := true;
        vp := min(vp,r)
    end;
send(tok,vp) to q0 with ~recp[q0];
receive(tok,r) from q0;
vp := min(vp,r);
if vp=p then state := leader else state := lost;
forall q in Neighp, q≠q0 do send(tok,vp) to q
end
```

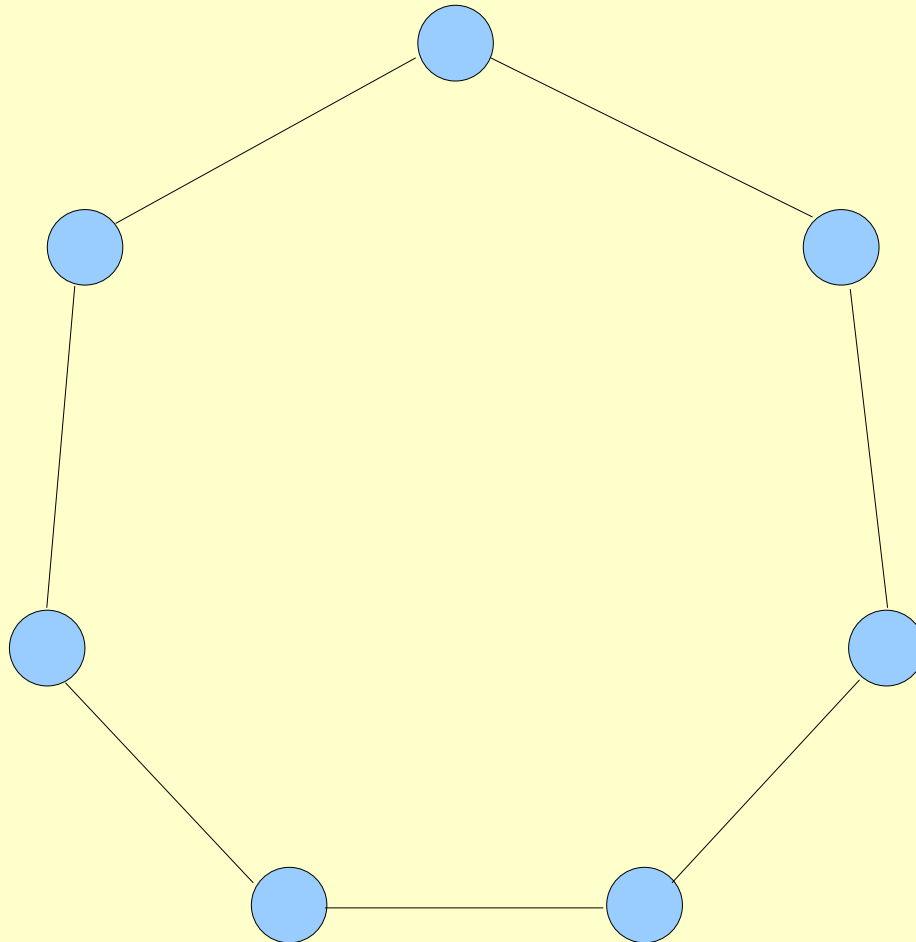
## Zprávy

4x (n-1) ... -?-> ... 3x (n-1)



# Election on ring

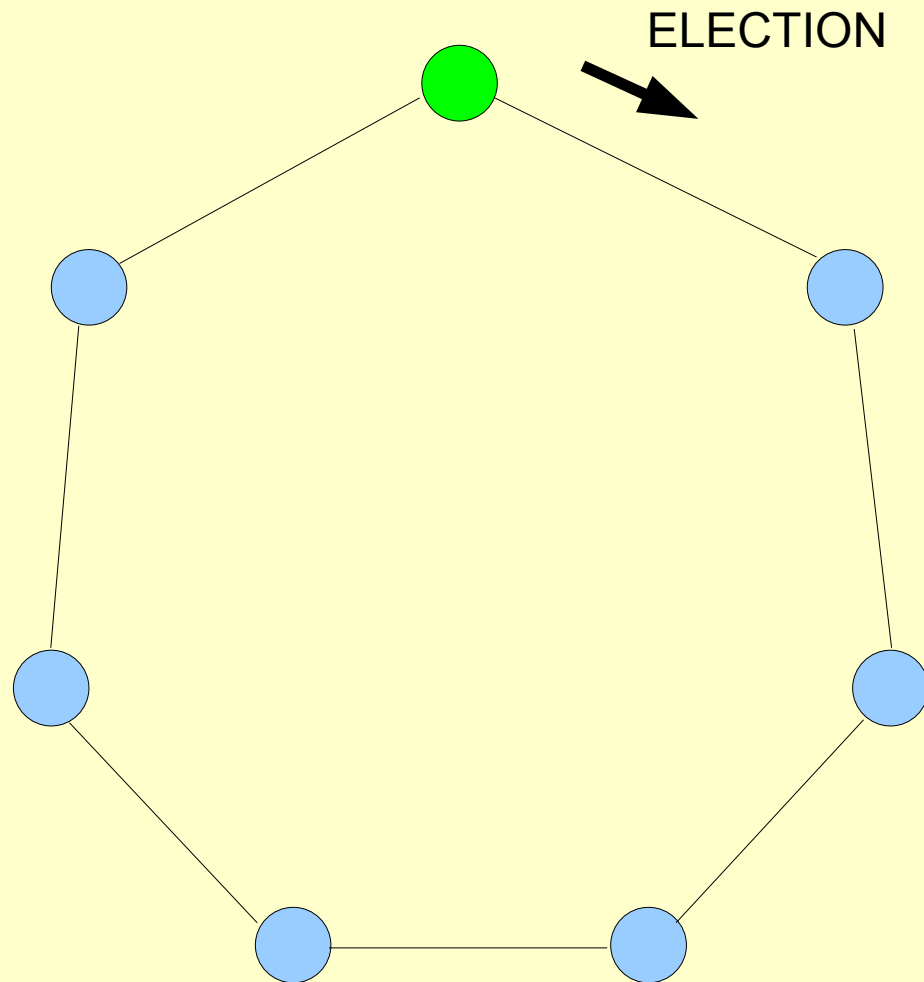
## Chang - Roberts





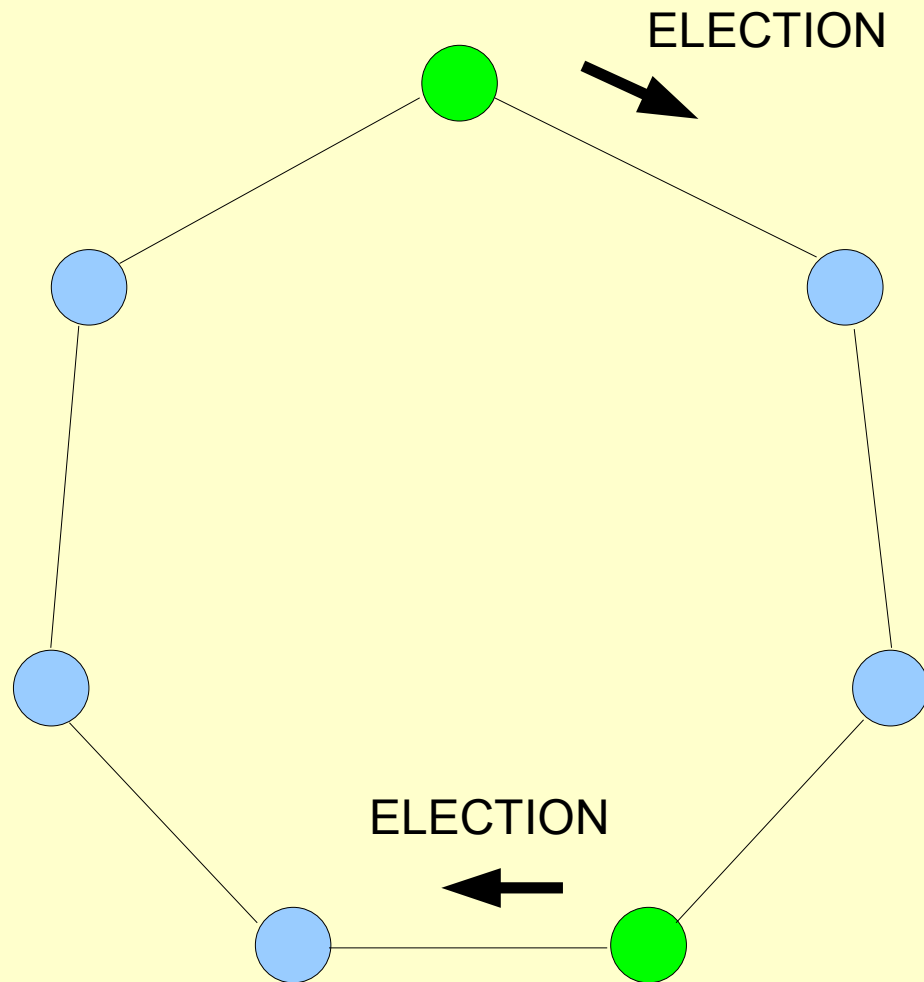
# Election on ring

## Chang - Roberts



# Election on ring

## Chang - Roberts



# Election on ring

## Chang - Roberts

```
var   Voting;  
      Coordinator;
```

```
begin  
    Voting:=F; Coordinator:=0  
end
```

{ inicializace }

```
when decision INITIATE_ELECTION do  
    begin  
        Voting:=T;  
        sendl ELECTION(i)  
    end
```

{ rozhodnutí volit }



# Election on ring

```
when received ELECTION(j) do
  begin
    if j>i then
      begin
        sendl ELECTION(j);
        Voting:=T
      end;
    if j<i and not Voting then
      begin
        sendl ELECTION(MyNumber);
        Voting:=T
      end;
    if j=i then
      begin
        sendl ELECTED(i)
      end
    end
  end
end
```

{ příjem zprávy ELECTION }



# Election on ring

```
when received ELECTED(j) do
  begin
    Coordinator:=j;
    Voting:=F;
    if j<>i then sendl ELECTED(j)
  end
```

{ příjem zprávy ELECTED }

## Zprávy

$(n-1)$  – min

$0.5n(n-1)$  – max

$O(n \log n)$  - avg



# Election on ring

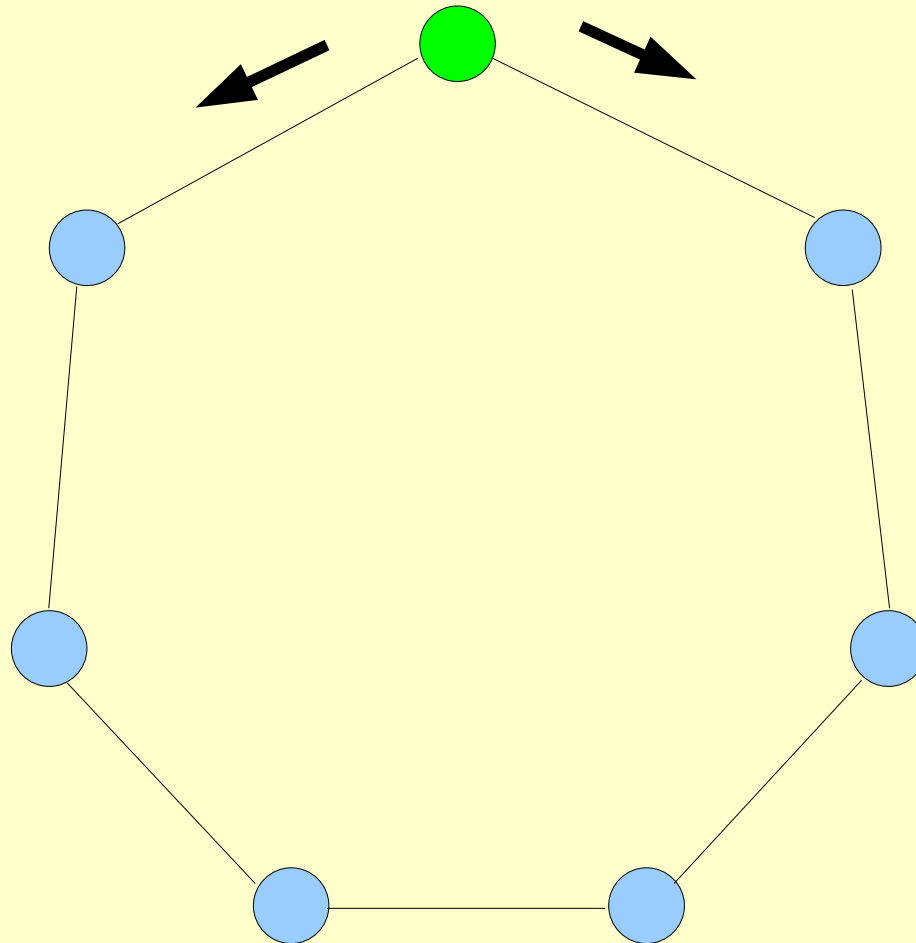
Chang - Roberts

```
var statep;
begin
  if p is initiator then
    begin
      statep := cand; send(tok,p) to Nextp;
      repeat receive(tok,q);
        if q=p then statep := leader
        else if q<p then
          begin if statep=cand then statep := lost;
            send(tok,q) to Nextp
          end
        until statep=leader
      end
    end
  else
    repeat receive(tok,q); send(tok,q) to Nextp;
      if statep=sleep then statep := lost
    until false
```



# Election on ring

Hirshberg - Sinclair



# Election on ring

## Hirshberg - Sinclair

**begin**

Nresp := 0; RespOK := T

**end**

{ inicializace }

**when decision** INITIATE\_ELECTION **do**

**begin**

State := CANDIDATE;

lmax := 1;

**while** State=CANDIDATE **do**

**begin**

Nresp := 0;

RespOK := T;

**sendlr** CANDIDATURE(i,0,lmax); // i – id, 0 – vzd., lmax - hloubka

**wait** NResp=2;

**if not** RespOK **then** State := LOST;

lmax := 2\*lmax

**end**

**end**

{ rozhodnutí volit }





# Election on ring

Hirshberg - Sinclair

```
when received CANDIDATURE(j,l,lmax) do
  begin
    if j<i then
      begin
        respond RESPONSE(F,j);
        if State=NOT_INVOLVED then INITIATE_ELECTION
      end;
    if j>i then
      begin
        State := LOST;
        l := l+1;
        if l<lmax then pass CANDIDATURE(j,l,lmax)
        else respond RESPONSE(T,j)
      end;
    if j=i then
      begin
        if State<>ELECTED then State:=ELECTED;
        Winner := i;
        pass ELECTED(i)
      end
    end
  end
end
```

{ příjem zprávy CANDIDATURE }



# Election on ring

## Hirshberg - Sinclair

```
when received RESPONSE(r,j) do
  if j=i then
    begin
      Nresp := NResp+1;
      RespOK := RestOK and r
    end
  else
    pass RESPONSE(r,j)
```

{ příjem zprávy RESPONSE }

```
when received ELECTED(j) do
  if Winner<>j then
    begin
      pass ELECTED(j);
      Winner := j;
      State := NOT_INVOLVED
    end
```

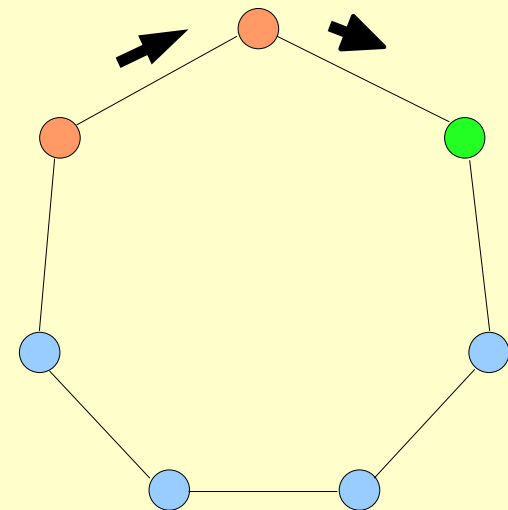
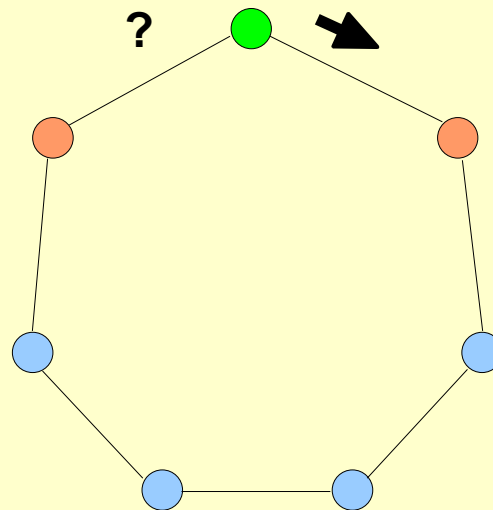
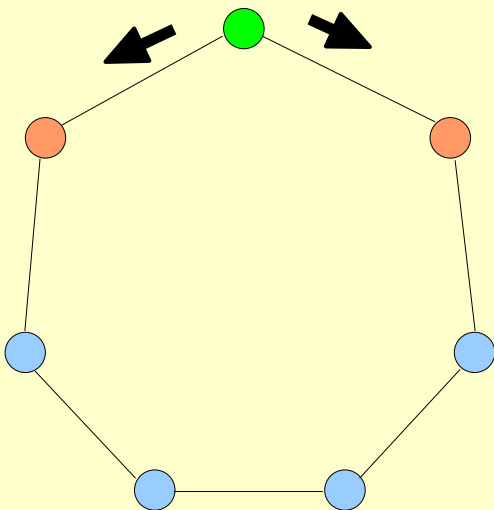
{ příjem zprávy ELECTED }



# Election on ring

## Peterson/DKR

```
var cip      : P init p ;      {Current identity of p}  
    acnp    : P init undef ;  {Id of anticlockwise active neighbor}  
    winp    : P init undef ;  {Id of winner}  
    statep  : (active, passive, leader, lost) init active ;
```



# Election on ring

## Peterson/DKR

```
begin if  $p$  is initiator then  $state_p := active$  else  $state_p := passive$  ;  
  while  $win_p = undef$  do begin { dokud neznáme vítěze }  
    if  $state_p = active$  then  
      begin send  $\langle one, ci_p \rangle$  ; receive  $\langle one, q \rangle$  ;  $acn_p := q$  ;  
        if  $acn_p = ci_p$  then {  $acn_p$  je minimum }  
          begin send  $\langle smal, acn_p \rangle$  ;  $win_p := acn_p$  ; receive  $\langle small, q \rangle$  ; end;  
        else {  $acn_p$  je současné Id suseda }  
          begin send  $\langle two, acn_p \rangle$  ; receive  $\langle two, q \rangle$  ;  
            if  $acn_p < ci_p$  and  $acn_p < q$   
              then  $ci_p := acn_p$   
              else  $state_p := passive$   
            end  
          end  
        end  
      end  
    else {  $state_p = passive$  }  
      begin receive  $\langle one, q \rangle$  ; send  $\langle one, q \rangle$  ;  
        receive  $m$  ; send  $m$  ; {  $m$  je  $\langle two, q \rangle$  nebo  $\langle smal, q \rangle$  }  
        if  $m$  is a  $\langle small, q \rangle$  message then  $win_p := q$   
      end  
    end  
  end  
  if  $p = win_p$  then  $state_p := leader$  else  $state_p := lost$   
end
```

